EE 508
Lecture 26

Nonideal Effects in Switched Capacitor Circuits

Noise

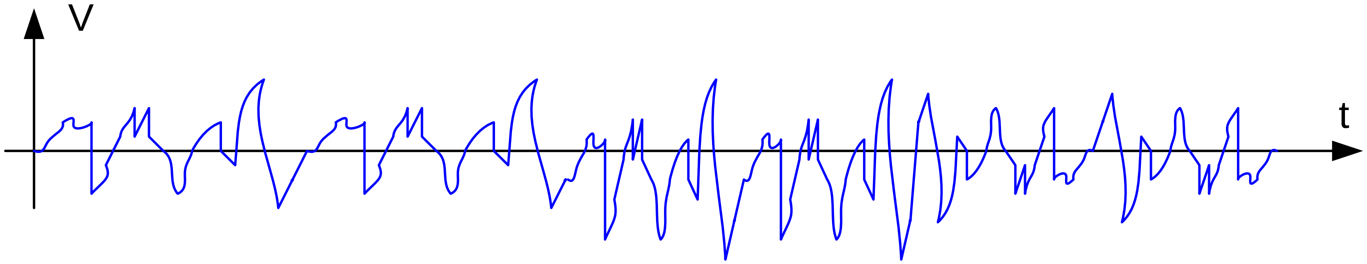
Switched-Resistor Filters

Other Integrators

Nonideal Effects in Switched Capacitor Circuits

- Parasitic Capacitances
- Op Amp Affects
- Charge Injection
- Aliasing
- Redundant Switch Removal
- Matching
- Noise

Noise in Linear Systems



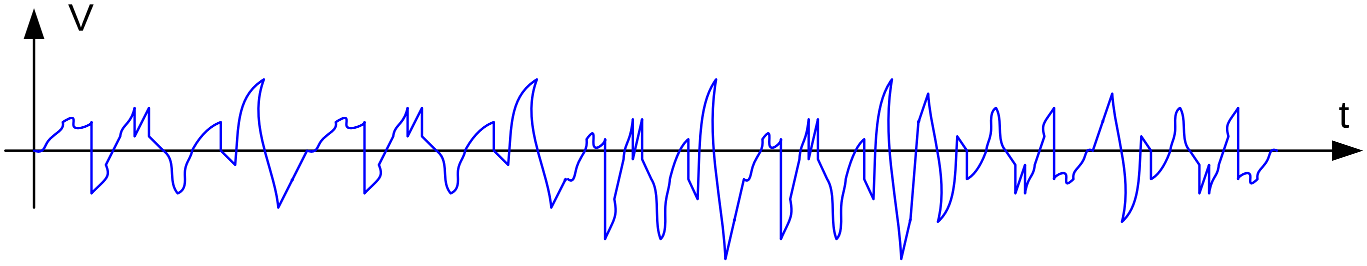
Noise in continuous-time systems

$$v_{RMS} = \sqrt{\lim_{T_x \to \infty}\left(\frac{1}{T_X}\int_0^{T_x} v_n^2(t)dt\right)}$$

At the design phase, we do not know what the time-domain noise characteristics will be

$$v_{RMS} = E\left(\sqrt{\lim_{T \to \infty}\left(\frac{1}{T}\int_0^{T} V^2(t)dt\right)}\right)$$

Noise in Linear Systems



Noise often characterized by the spectral density S

$$v_{RMS} = \sqrt{\left(\int_0^\infty S(f)\,df\right)}$$

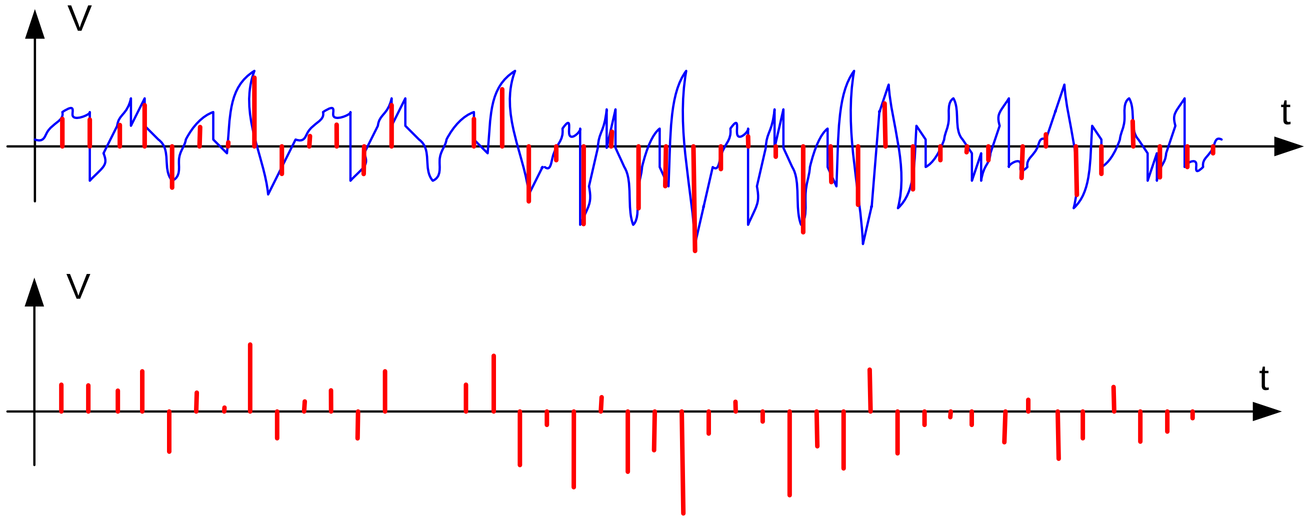
$$\int_{f=0}^{\infty} S(f)\,df = \lim_{f_x \to \infty} \int_{f=0}^{f_x} S(f)\,df$$

Thus

$$\sqrt{\lim_{T_x \to \infty}\left(\frac{1}{T_X}\int_0^{T_x} v_n^2(t)\,dt\right)} = \sqrt{\left(\int_0^\infty S(f)\,df\right)} = \sqrt{\lim_{f_x \to \infty} \int_{f=0}^{f_x} S(f)\,df}$$

Noise in Linear Systems

Noise in discrete-time linear systems

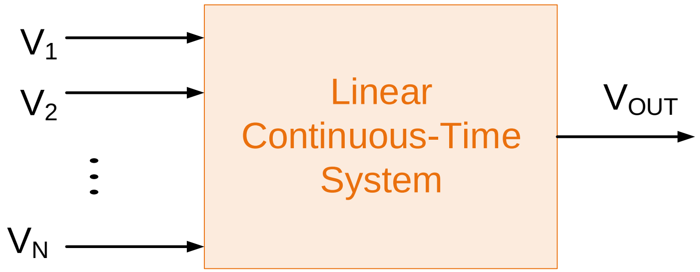


$$\vec{V}(kT) = \left\langle \mathcal{V}_n(mT) \right\rangle_{m=0}^{\infty}$$

$$\vec{V}_{RMS} = \sqrt{\lim_{M\to\infty}\left(\frac{1}{M}\sum_{k=0}^{M} \mathcal{V}_n^2(kT)\right)}$$

$$\sigma\left(\mathcal{V}_n(kT)\right) = \mathcal{V}_{RMS}$$

$$\mathcal{V}_{RMS} = E\left(\sqrt{\lim_{T\to\infty}\left(\frac{1}{T}\int_0^T V^2(t)dt\right)}\right)$$



$$V_{OUT}(s) = \sum_{i=1}^{N} T_i(s) V_i(s)$$

$$S_{OUT} = \sum_{i=1}^{N} S_i \bullet \left| T_i(j\omega) \right|^2$$

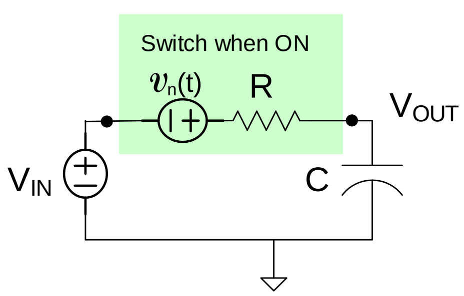
$$\mathcal{V}_{OUT\_RMS} = \sqrt{\int_{f=0}^{\infty} S_{OUT} df} = \sqrt{\int_{f=0}^{\infty} \sum_{i=1}^{N} S_i \bullet \left| T_i(j\omega) \right|^2 df}$$

Noise

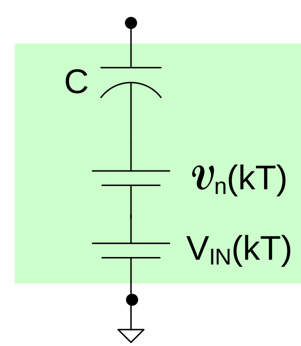
- Capacitors do not have any noise source
- Switches contribute thermal noise
- Noise due to switches looks like "capacitive" noise

$$V_{RMS} = \sqrt{\frac{kT}{C}}$$

Consider a capacitor that is sampling an input signal V_{IN}

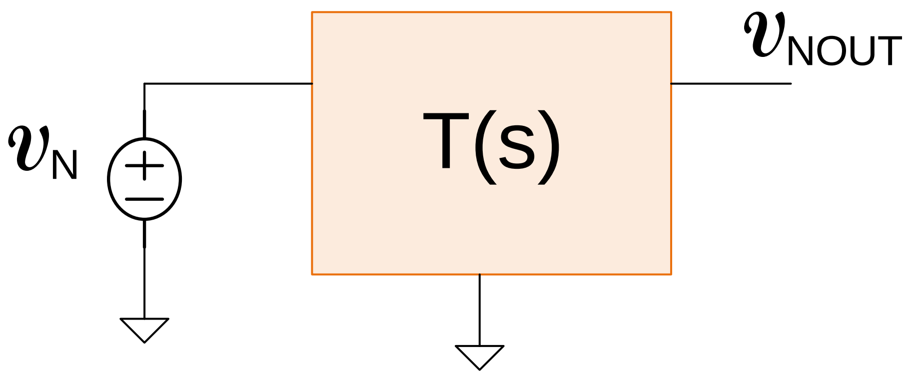


Track mode



Hold mode

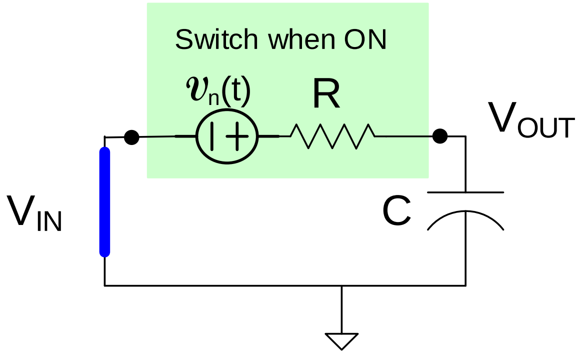
Noise review



Component of spectral density at output due to any noise source with spectral density S_X given by

$$S(\omega) = S_X \left| T(j\omega) \right|^2$$

Noise during sampling phase



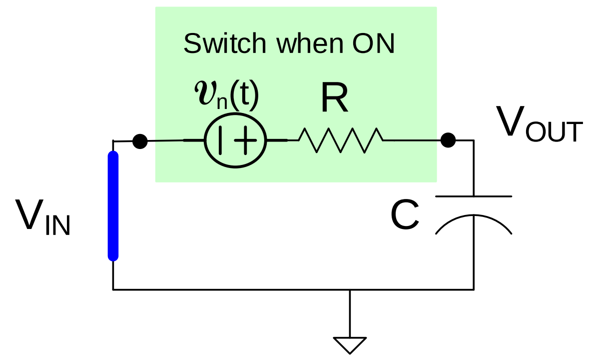
$$S(\omega) = S_X \left| T(j\omega) \right|^2$$

$$T(s) = \frac{1}{1 + RCs}$$

$$S_{Vn} = 4kTR$$

$$S_{VOUT} = 4kTR\left(\frac{1}{1+(RC\omega)^2}\right)$$

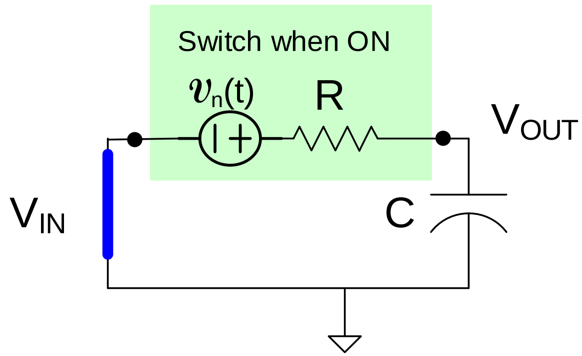
Noise during sampling phase



$$S_{VOUT} = 4kTR\left(\frac{1}{1+(RC\omega)^2}\right)$$

$$v_{n_{RMS}} = \sqrt{\int_{f=0}^{\infty} S_{V_{OUT}}\, df} = \sqrt{\int_{f=0}^{\infty} \frac{4kTR}{1+\omega^2R^2C^2}\, df}$$

Noise during sampling phase

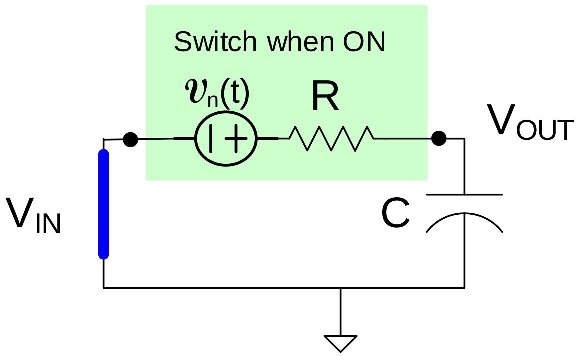


$$v_{n_{RMS}} = \sqrt{\int_{f=0}^{\infty} S_{V_{OUT}}\, df} = \sqrt{\int_{f=0}^{\infty} \frac{4kTR}{1+\omega^2 R^2 C^2}\, df}$$

It can be shown that this integral is independent of R and is given by

$$v_{n_{RMS}} = \sqrt{\int_{f=0}^{\infty} S_{V_{OUT}}\, df} = \sqrt{\frac{kT}{C}}$$

Noise during sampling phase



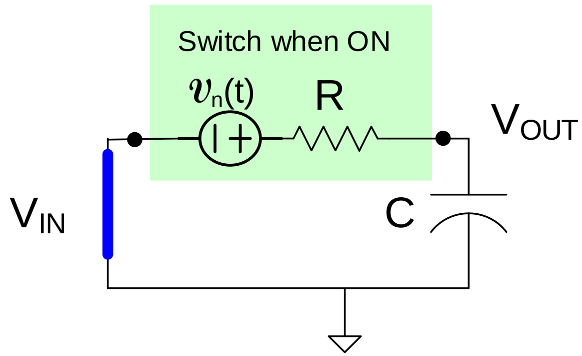
$$v_{n_{RMS}} = \sqrt{\frac{kT}{C}}$$

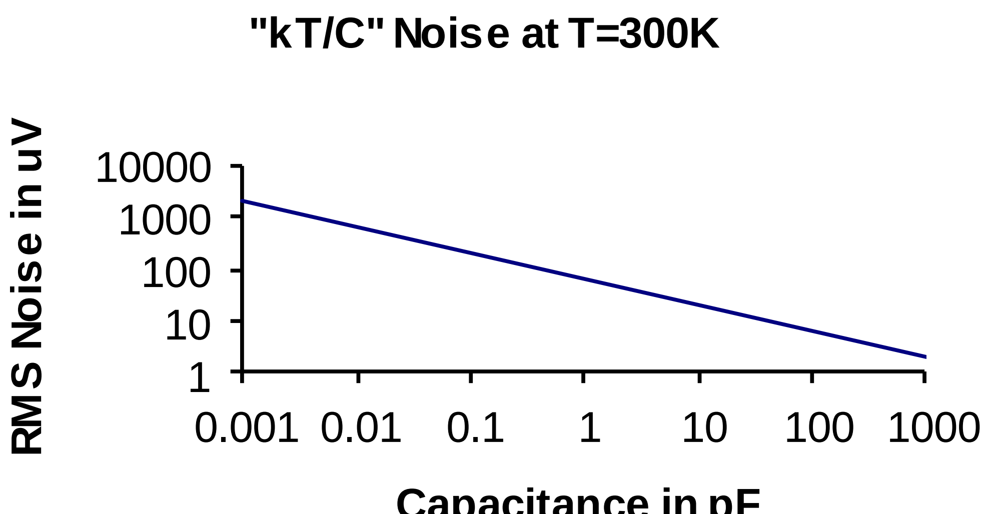
RMS noise voltage on C is independent of the state of the switch

So sampled RMS noise voltage should be same as instantaneous RMS voltage

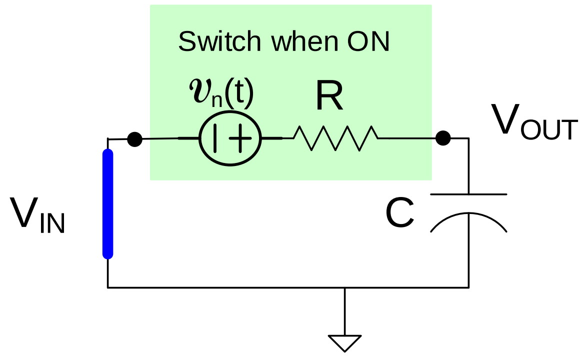
Highly temperature dependent

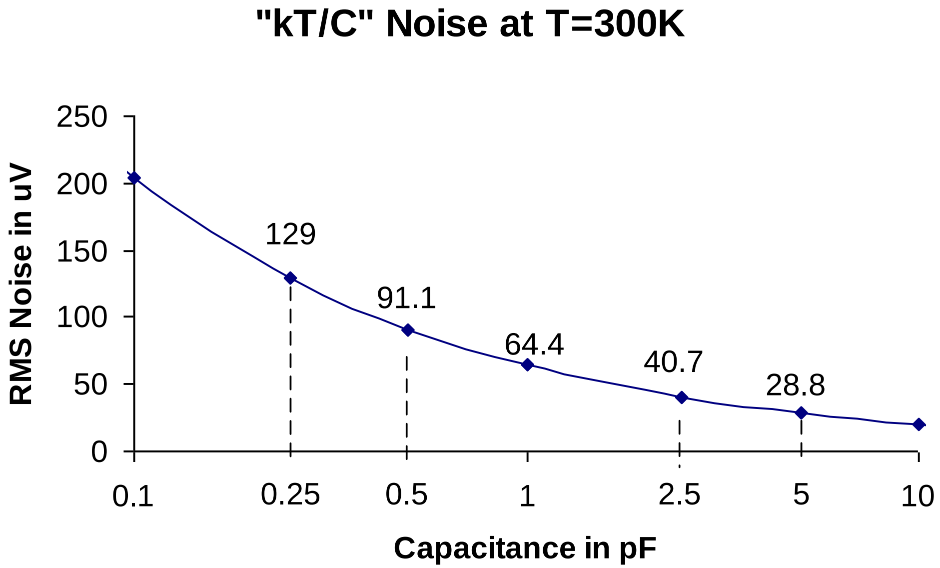
Noise during sampling phase





Noise during sampling phase





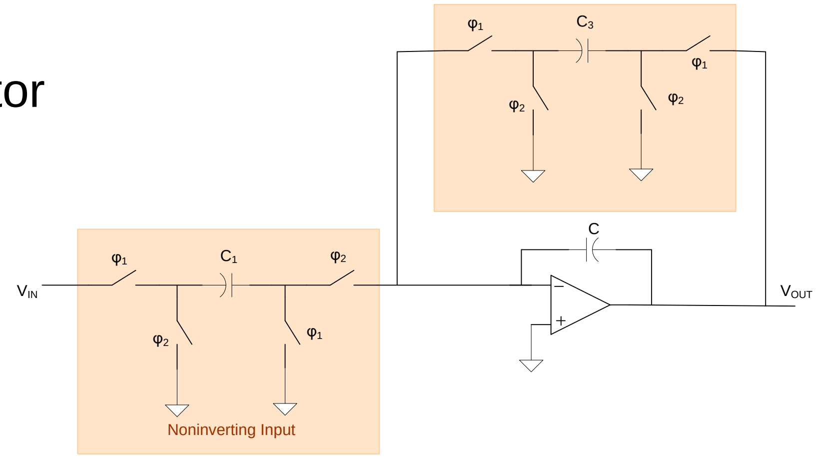
Noise

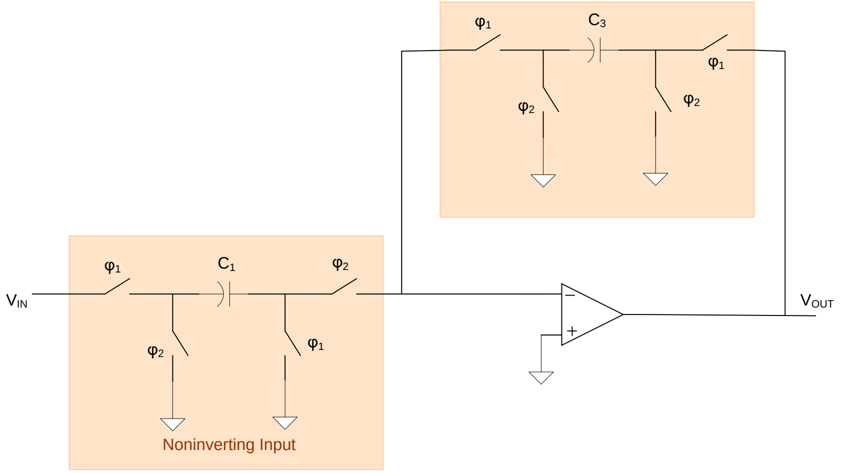
- Capacitors do not have any noise source
- Switches contribute thermal noise
- Noise due to switches looks like “capacitive” noise $V_{RMS} = \sqrt{\frac{kT}{C}}$

Be careful with calculating noise in SC circuits !

Switched Capacitor Amplifiers

Elimination of the Integration Capacitor

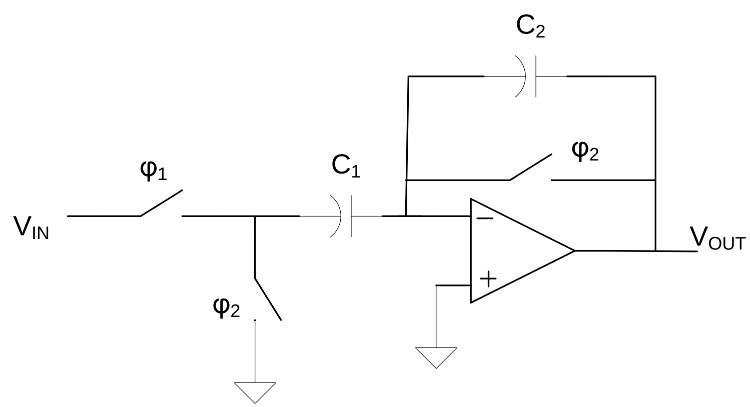


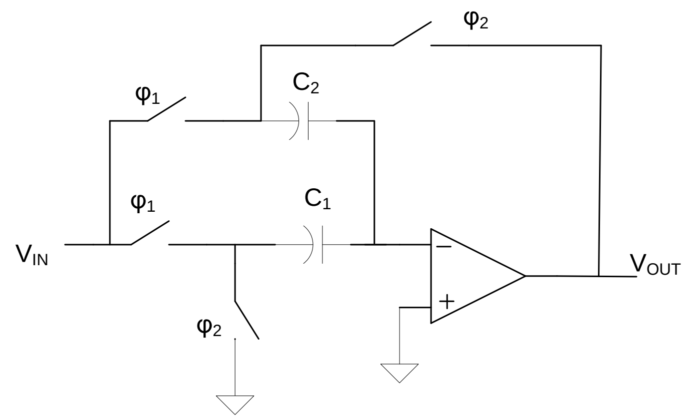


What happens if the integration capacitor is eliminated?

- Serves as a SC amplifier with gain of $A_V = C_1/C_2$
- SC amplifiers and SC summing amplifiers are widely used in filter and non-filter applications

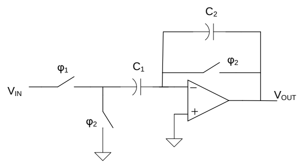
Switched Capacitor Amplifiers



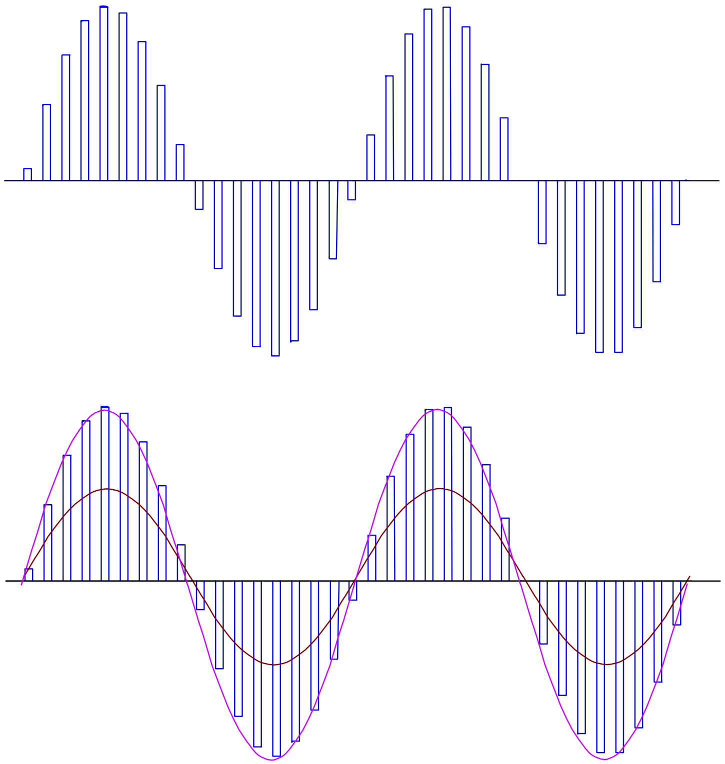


- Summing, Differencing, Inverting, and Noninverting SC Amplifiers Widely Used
- Significant reduction in switches from what we started with by eliminating C in SC integrator
- Must be stray insensitive in most applications
- Outputs valid only during one phase

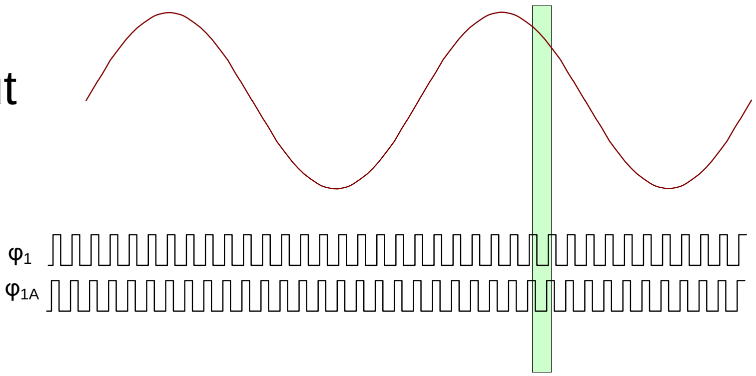
Switched Capacitor Amplifiers



Output



Input



Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

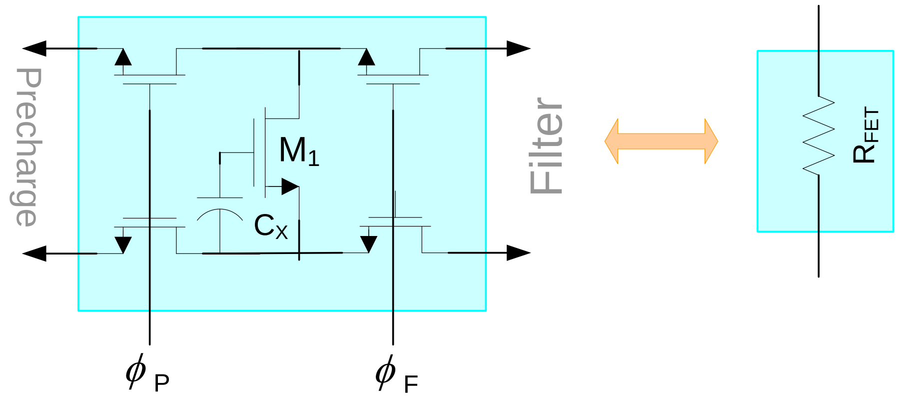
Sometimes termed "current mode"

- Switched Capacitor
- Switched Resistor

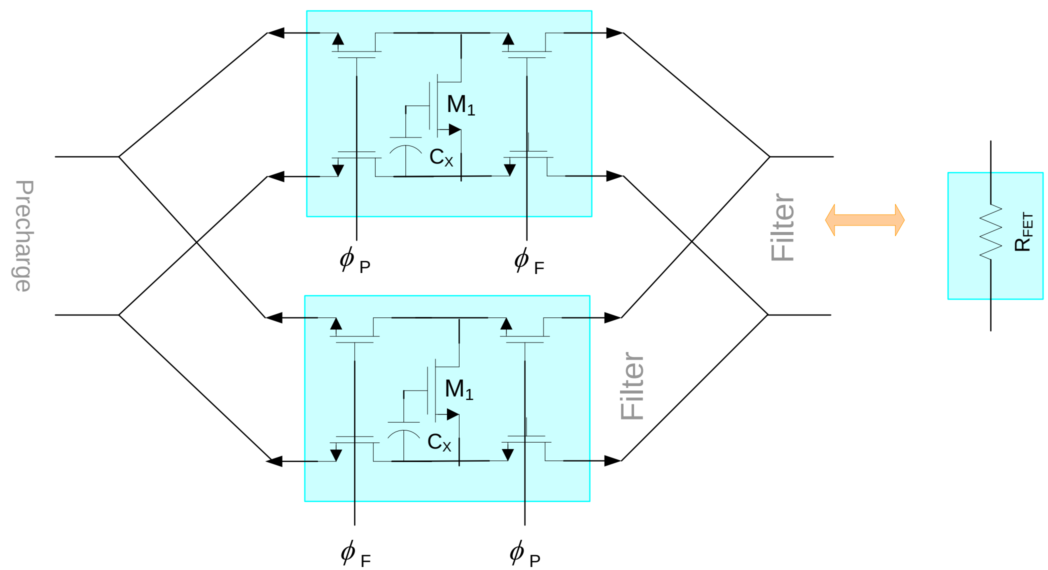
Will discuss later

- Other Structures

Switched-Resistor Voltage Mode Integrators



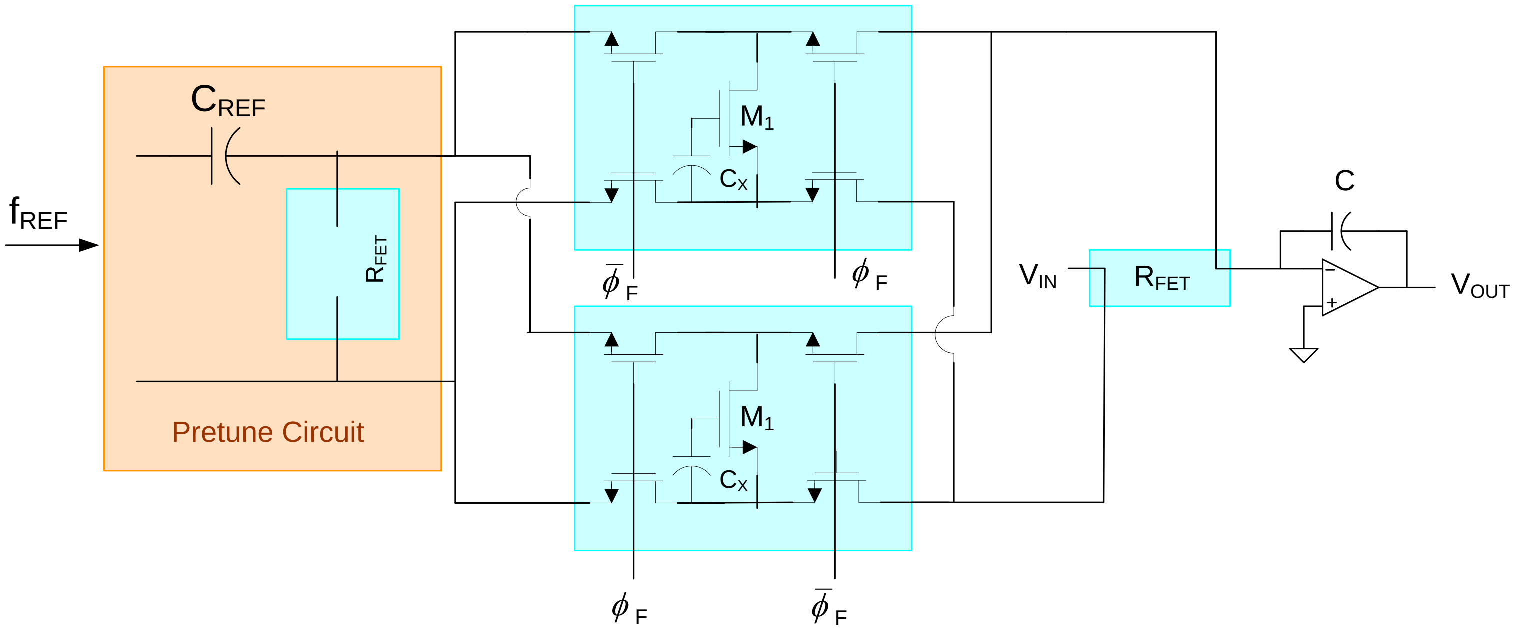
Observe that if a triode-region MOS device is switched between a precharge circuit and a filter circuit (or integrator) and V_{GS} is held constant, It will behave as a resistor while in the filter circuit



$$R_{FET} \cong \frac{L}{\mu C_{OX} W (V_{GS} - V_T)}$$

Observe that if two such circuits are switched between a precharge circuit and a filter circuit (or integrator) and V_{GS} is held constant, It will behave as a resistor in the filter circuit at all times

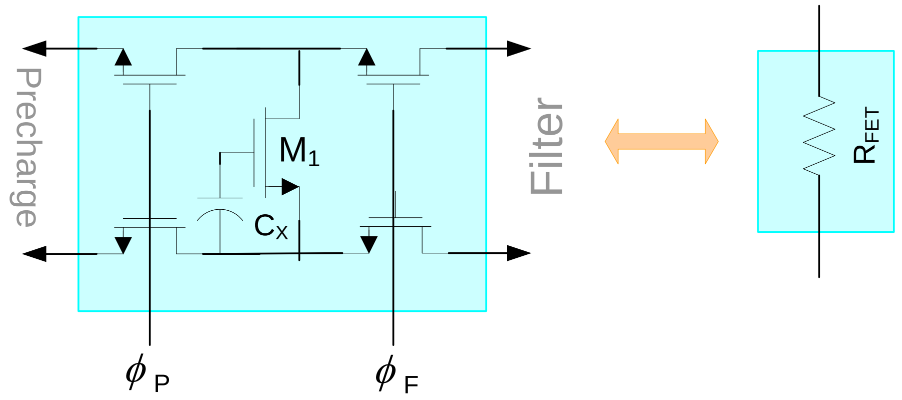
Switched-Resistor Voltage Mode Integrators



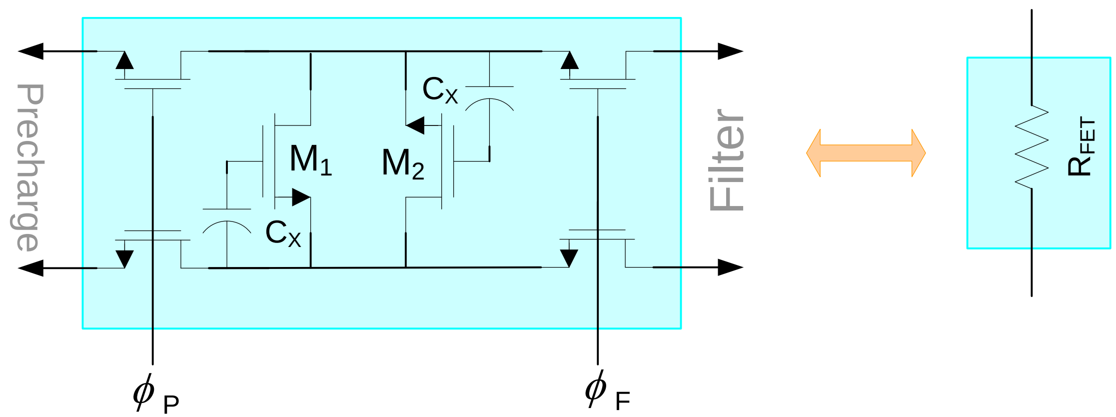
Switched-resistor integrator

- Clock frequency need only be fast enough to prevent droop on C_X
- Minor overlap or non-overlap of clock plays minimal role in integrator performance
- Switched-resistors can be used for integrator resistor or to replace all resistors in any filter
- Pretune circuit can accurately establish $R_{FET}C_{REF}$ product proportional to f_{REF}
- $R_{FET}C$ product is given by $R_{FET}C = R_{FET}C\frac{C_{REF}}{C_{REF}} = [R_{FET}C_{REF}]\bullet\left[\frac{C}{C_{REF}}\right]$ and is thus accurately controlled

Switched-Resistor Voltage Mode Integrators

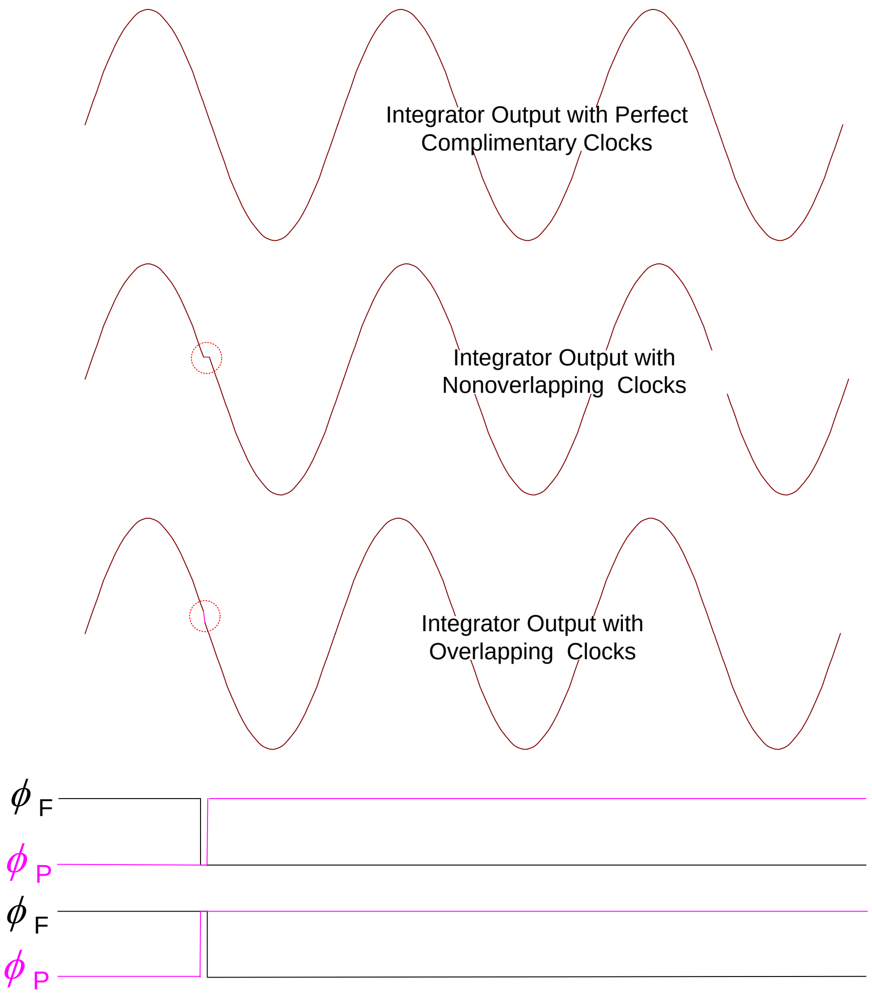


There are some modest nonlinearities in this MOSFET when operating in the triode region



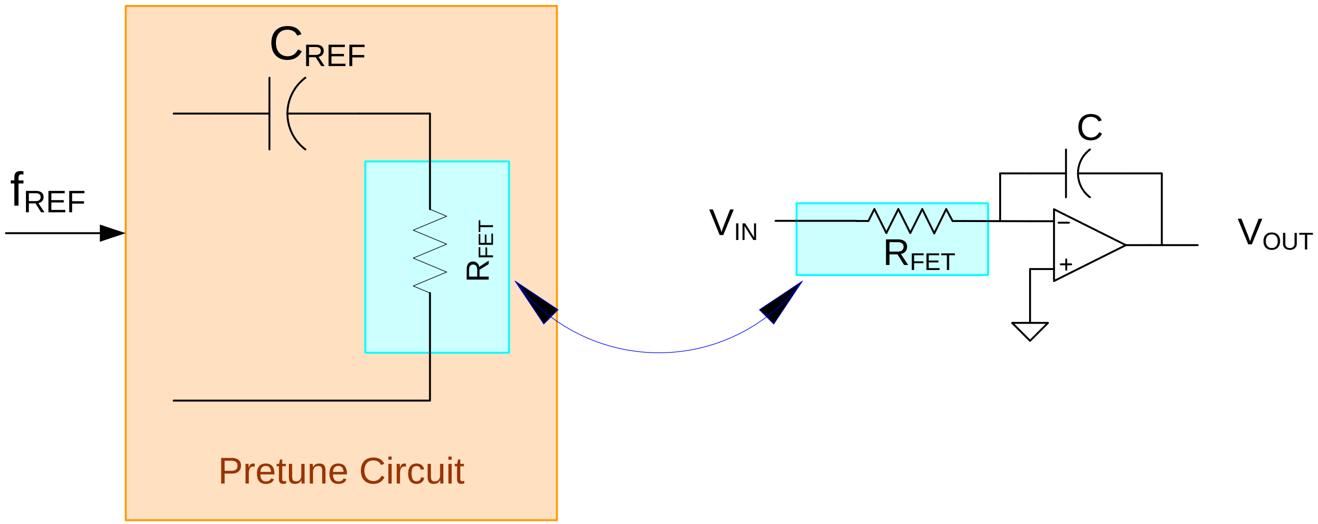
- Significant improvement in linearity by cross-coupling a pair of triode region resistors
- Perfectly cancels nonlinearities if square law model is valid for M_1 and M_2
- Only modest additional complexity in the Precharge circuit

Switched-Resistor Voltage Mode Integrators



- Aberrations are very small, occur very infrequently, and are further filtered
- Play almost no role on performance of integrator or filter

Switched-Resistor Voltage Mode Integrators



Switched-resistor integrator

- Accurate CR_{FET} products is possible
- Area reduced compared to Active RC structure because R_{FET} small
- Single pretune circuit can be used to "calibrate" large number of resistors
- Clock frequency not fast and not critical (but accuracy of f_{REF} is important)
- Since resistors are memoryless elements, no transients associated with switching
- Since filter is a feedback structure, speed limited by BW of op amp

Voltage Mode Integrators

- Active RC (Feedback-based)
- MOSFET-C (Feedback-based)
- OTA-C
- TA-C

} Sometimes termed "current mode"

- Switched Capacitor (Feedback-based)
- Switched Resistor (Feedback-based)

} Discrete Time

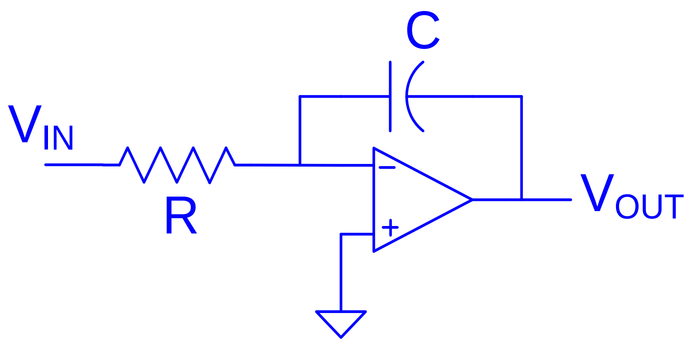
- Other Structures

Have introduced a basic voltage-mode integrators in each of these approaches

All of these structures have applications where they are useful

Performance of feedback-based structures limited by Op Amp BW

Variants of basic inverting integrator have been considered



Basic Miller Integrator

- Active RC
- MOSFET-C
- OTA-C
- g_m-C
- Switched-Capacitor
- Switched-Resistor

Performance of all is limited by GB of Operational Amplifiers

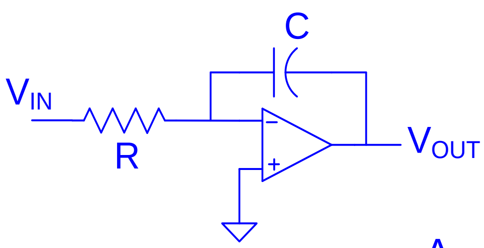
How can integrator performance be improved?

- Better op amps
- Better Integrator Architectures

How can the performance of integrator structures be compared?

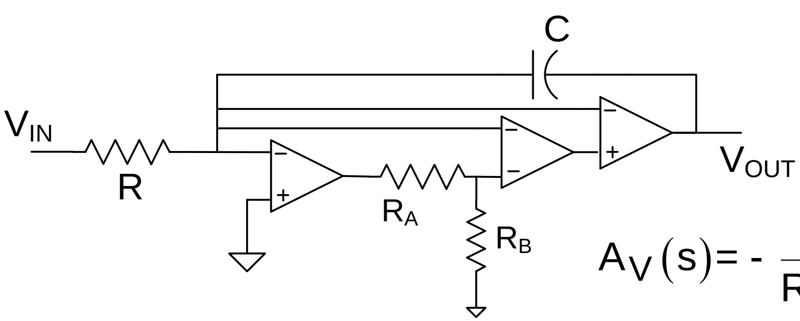
Need metric for comparing integrator performance

Are there other integrators in the basic classes that have been considered?



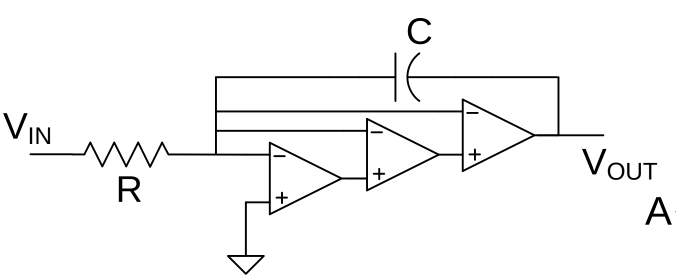
$$A_V(s) = -\frac{1}{RCs}$$

Miller Inverting



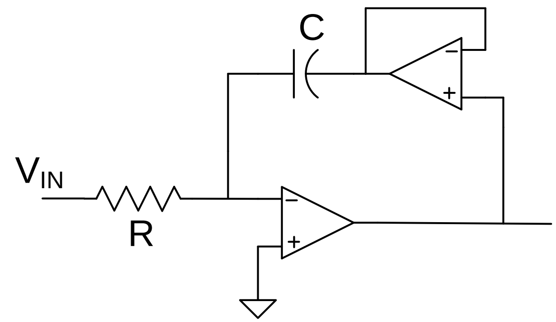
$$A_V(s) = -\frac{1}{RCs}$$

Zero Second Derivative Inverting



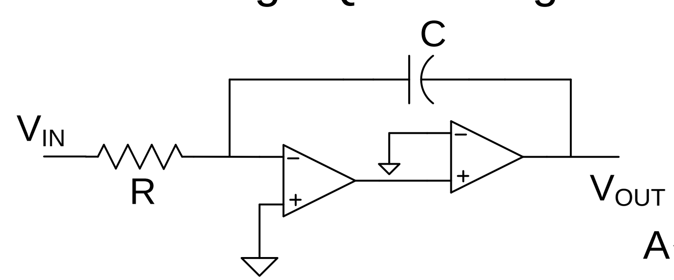
$$A_V(s) = -\frac{1}{RCs}$$

Zero Second Derivative Inverting



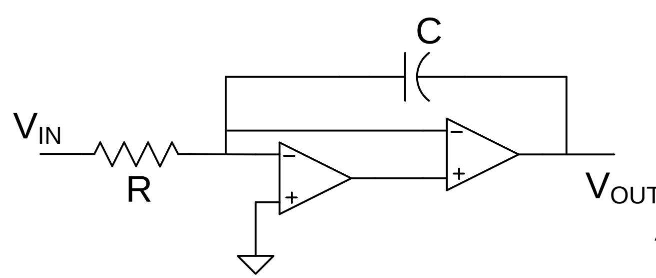
$$A_V(s) = -\frac{1}{RCs}$$

High-Q Inverting



$$A_V(s) = -\frac{1}{RCs}$$

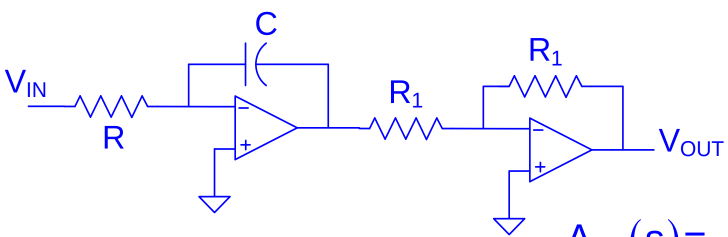
Cascaded Inverting



$$A_V(s) = -\frac{1}{RCs}$$

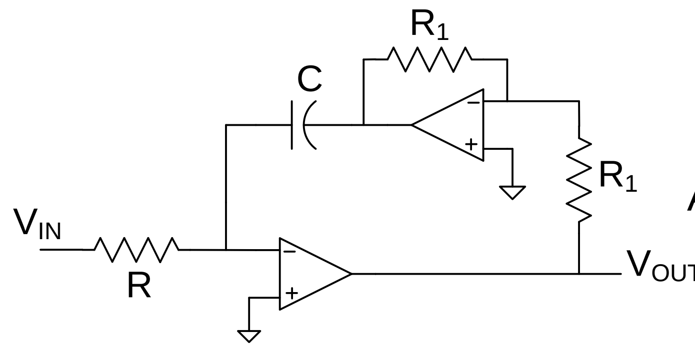
Zero Sensitivity Inverting

Are there other integrators in the basic classes that have been considered?



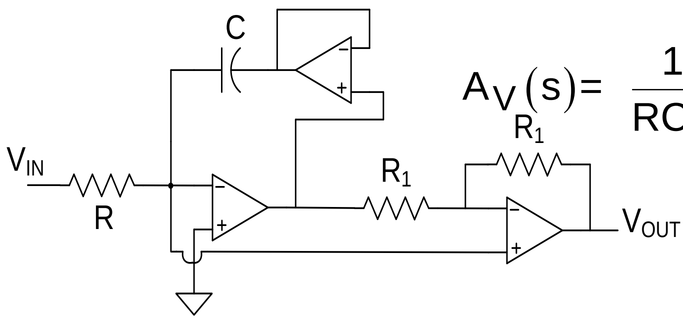
Miller Noninverting

$$A_V(s) = \frac{1}{RCs}$$



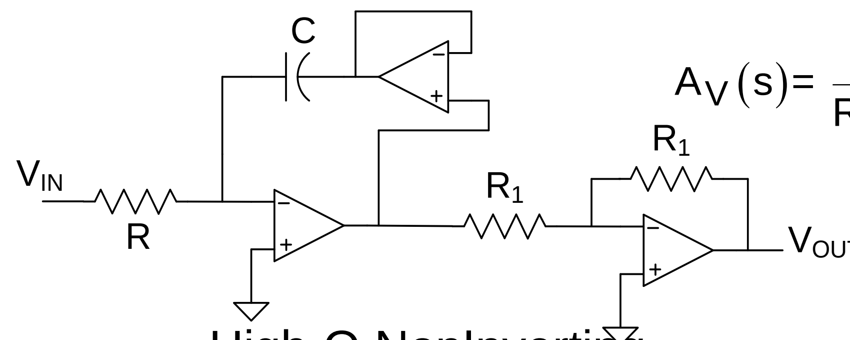
$$A_V(s) = \frac{1}{RCs}$$

Phase Lead Integrator



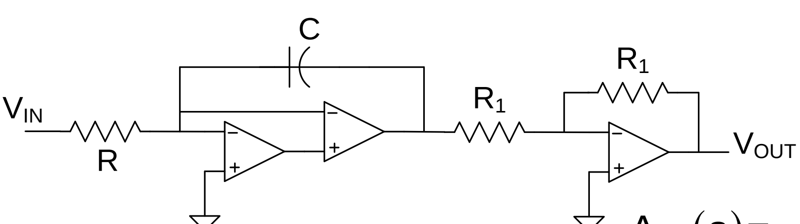
$$A_V(s) = \frac{1}{RCs}$$

Modified High-Q Noninverting



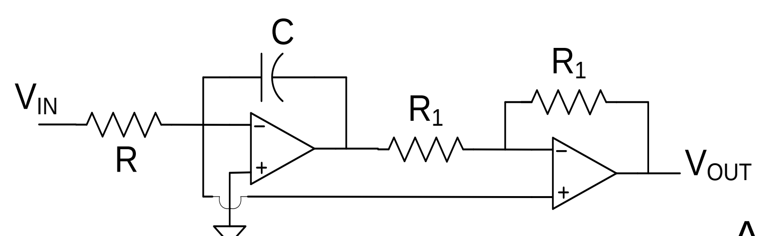
$$A_V(s) = \frac{1}{RCs}$$

High-Q NonInverting



$$A_V(s) = \frac{1}{RCs}$$

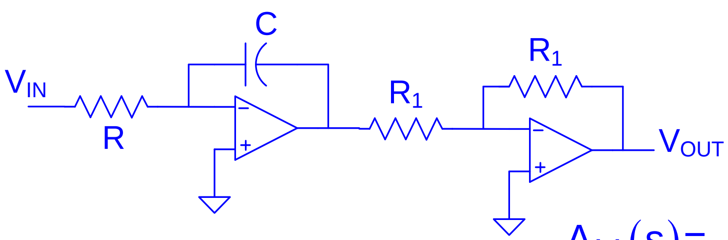
Zero Sensitivity Noninverting



$$A_V(s) = \frac{1}{RCs}$$

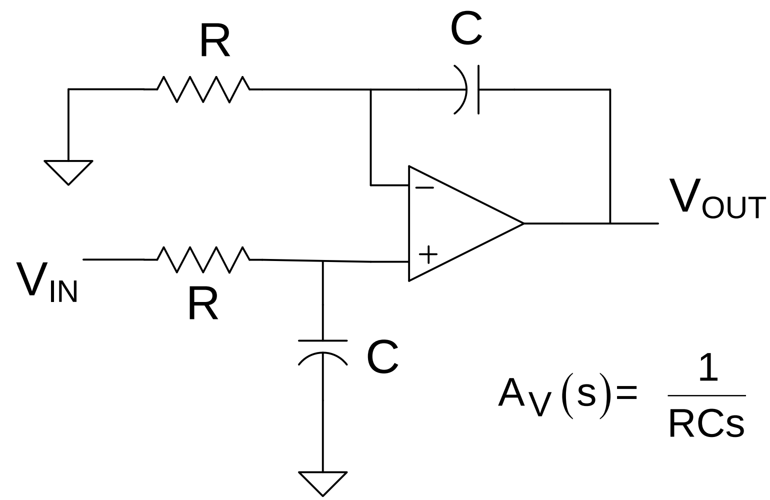
Modified Miller Noninverting

Are there other integrators in the basic classes that have been considered?



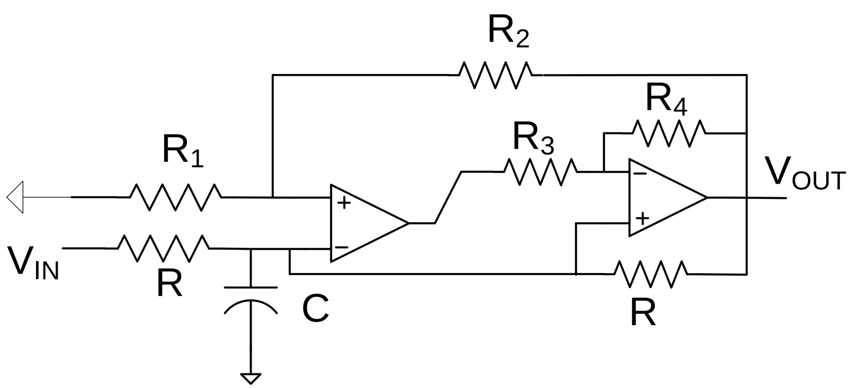
Miller Noninverting

$$A_V(s)=\frac{1}{RCs}$$



$$A_V(s)=\frac{1}{RCs}$$

Balanced Time Constant Noninverting



Zero Sensitivity Noninverting

$$A_V(s)=\frac{2}{RCs}$$

If R_1=R_2 and R_3=R_4

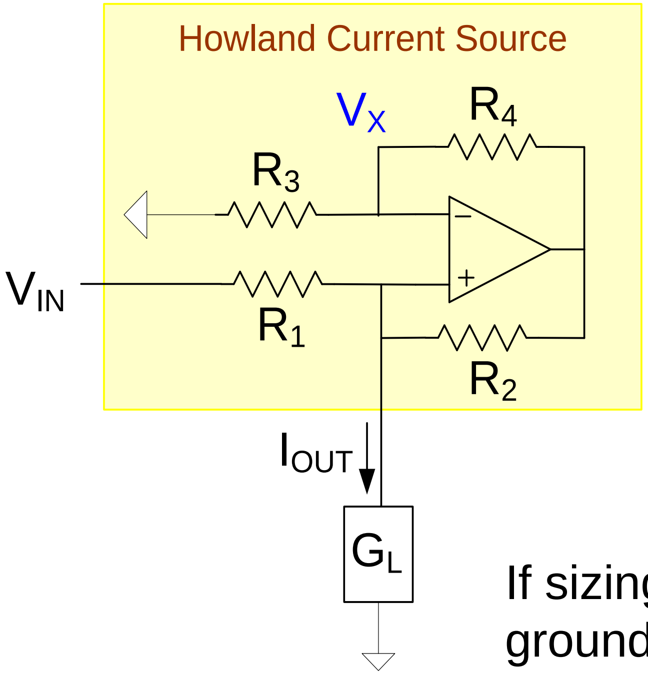
(note this has a grounded integrating capacitor!)

Stay Safe and Stay Healthy !

End of Lecture 25

De Boo Integrator

Consider the Howland Current Source



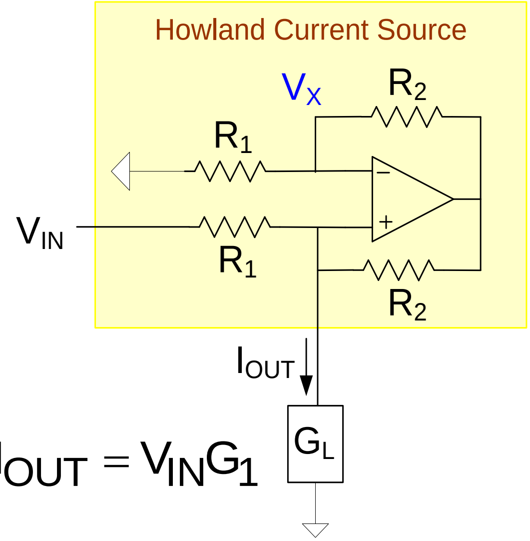
$$I_{OUT} = V_{IN}G_1 + \left[V_X \left(\frac{G_2G_3}{G_4} - G_1 \right) \right]$$

If resistors sized so that $G_1 = \frac{G_2G_3}{G_4}$

$$I_{OUT} = V_{IN}G_1$$

If sizing constraints are satisfied, behaves as a grounded constant-current source

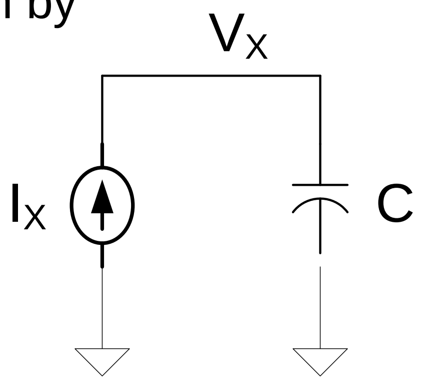
DeBoo Integrator



$I_{OUT} = V_{IN}G_1$

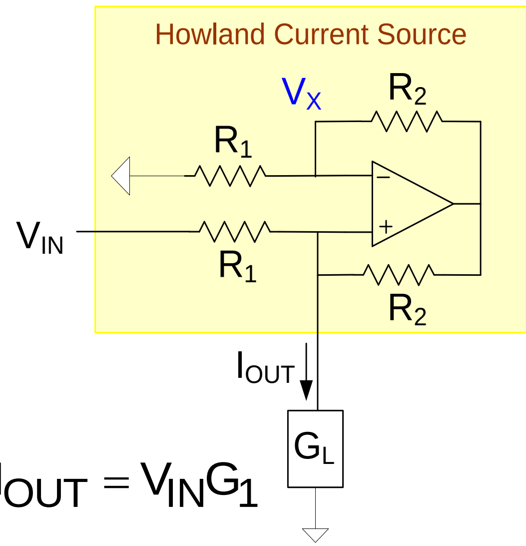
Observe that if a current source drives a grounded capacitor, then the nodal voltage on the capacitor is given by

$$V_X = I_X \frac{1}{sC}$$

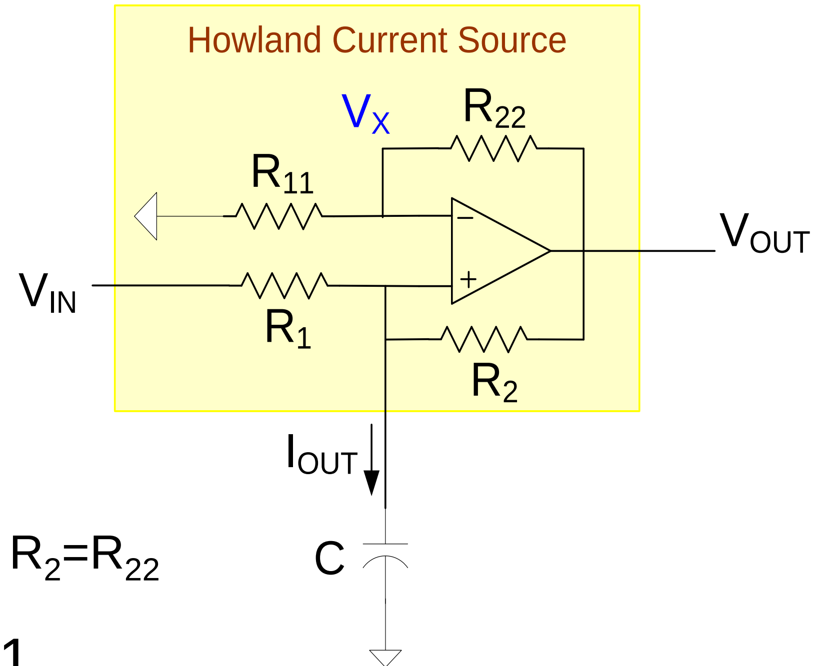


Thus, if we could make I_X proportional to V_{IN}, the voltage on the capacitor would be a weighted Integral of V_{IN}

De Boo Integrator



$$I_{OUT} = V_{IN}G_1$$



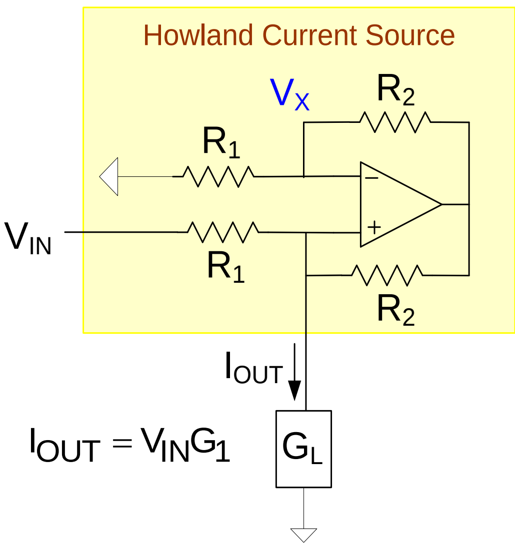
If $R_1 = R_{11}$ and $R_2 = R_{22}$

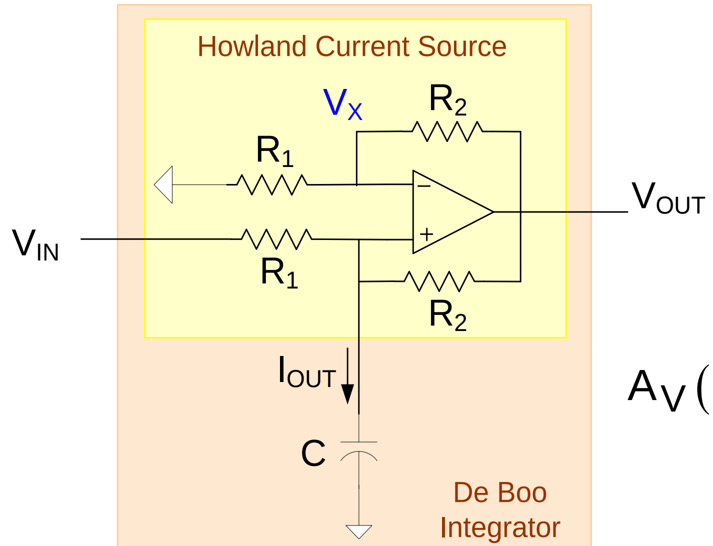
$$V_X = \frac{V_{IN}}{R_1}\frac{1}{sC}$$

$$V_{OUT} = V_X\left(1 + \frac{R_{22}}{R_{11}}\right) = \frac{V_{IN}}{R_1}\frac{1}{sC}\left(1 + \frac{R_{22}}{R_{11}}\right)$$

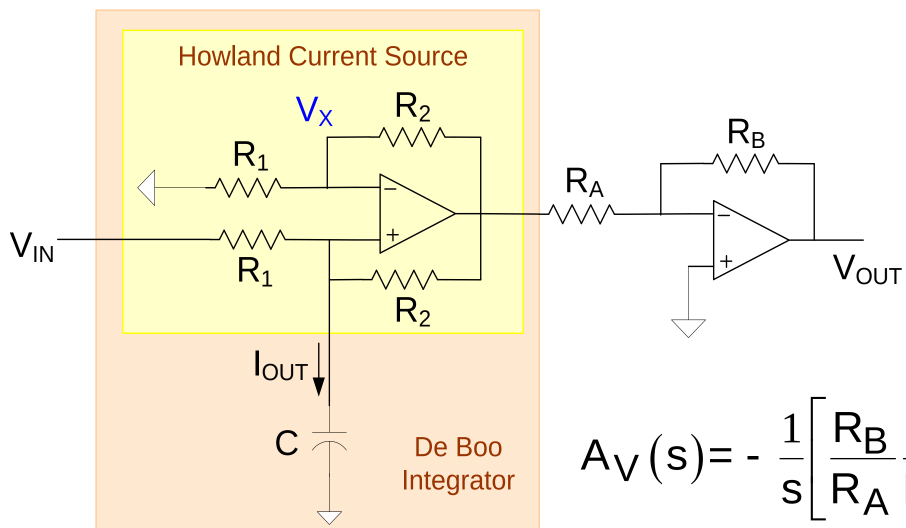
$$A_V(s) = \frac{1}{s}\left[\frac{1}{R_1 C}\left(1 + \frac{R_{22}}{R_{11}}\right)\right]$$

De Boo Integrator





$$A_V(s) = \frac{1}{s}\left[\frac{1}{R_1C}\left(1+\frac{R_{22}}{R_{11}}\right)\right]$$



$$A_V(s) = -\frac{1}{s}\left[\frac{R_B}{R_A}\frac{1}{R_1C}\left(1+\frac{R_{22}}{R_{11}}\right)\right]$$

Many different integrator architectures that ideally provide the same gain

Similar observations can be made for other classes of integrators

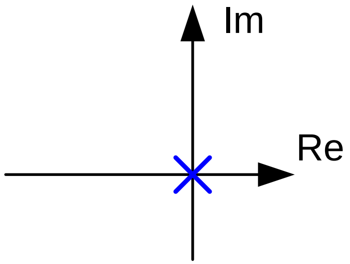
How can the performance of an integrator be characterized and how can integrators be compared?

How can the performance of an integrator be characterized and how can integrators be compared?

Consider Ideal Integrator Gain Function

$$A_V(s) = \frac{I_0}{s} \qquad A_V(j\omega) = \frac{I_0}{j\omega}$$

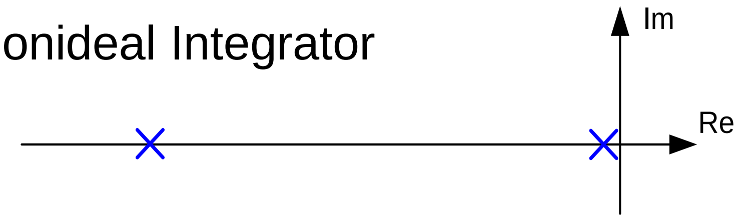
Ideal Integrator



Consider a nonideal integrator Gain Function

$$A_V(s) = \frac{\alpha I_{01}}{s+\alpha} A_{OO}(s)$$

Nonideal Integrator



Key characteristics of an ideal integrator:

- Magnitude of the gain at I_0=1
- Phase of integrator always 90^o
- Gain decreases with 1/ω

Are any of these properties more critical than others?

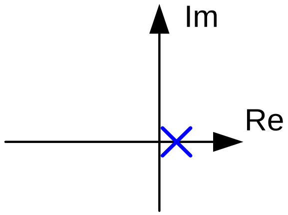
In many applications:

Key property of ideal integrator is a phase shift of 90^o at frequencies around I_0!

How can the performance of an integrator be characterized and how can integrators be compared?

Is stability of an integrator of concern?

Ideal Integrator



- Ideal integrator is not stable
- Integrator function is inherently ill-conditioned
- Integrator is almost never used open-loop
- Stability of integrator not of concern, stability of filter using integrator is of concern
- Some integrators may cause unstable filters, others may result in stable filters
- Instability in filter because desired poles move in RHP is of little concern since the filter performance would be unacceptable long before the stability became an issue
- Instability in filter due to parasitic poles is of concern but not a problem in most circuits